

LECTURE NO 14

Electrostatics

Topics

- Calculation of Coulomb force due to point charge
- Vector sum of forces

Numerical

Point charges 1 mC and -2 mC are located at $(3, 2, -1)$ and $(-1, -1, 4)$, respectively. Calculate the electric force on a 10-nC charge located at $(0, 3, 1)$ and the electric field intensity at that point.

Solution:

$$\begin{aligned}\mathbf{F} &= \sum_{k=1,2} \frac{QQ_k}{4\pi\epsilon_0 R^2} \mathbf{a}_R = \sum_{k=1,2} \frac{QQ_k(\mathbf{r} - \mathbf{r}_k)}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_k|^3} \\ &= \frac{Q}{4\pi\epsilon_0} \left\{ \frac{10^{-3}[(0, 3, 1) - (3, 2, -1)]}{|(0, 3, 1) - (3, 2, -1)|^3} - \frac{2 \cdot 10^{-3}[(0, 3, 1) - (-1, -1, 4)]}{|(0, 3, 1) - (-1, -1, 4)|^3} \right\} \\ &= \frac{10^{-3} \cdot 10 \cdot 10^{-9}}{4\pi \cdot \frac{10^{-9}}{36\pi}} \left[\frac{(-3, 1, 2)}{(9 + 1 + 4)^{3/2}} - \frac{2(1, 4, -3)}{(1 + 16 + 9)^{3/2}} \right] \\ &= 9 \cdot 10^{-2} \left[\frac{(-3, 1, 2)}{14\sqrt{14}} + \frac{(-2, -8, 6)}{26\sqrt{26}} \right] \\ \mathbf{F} &= -6.507\mathbf{a}_x - 3.817\mathbf{a}_y + 7.506\mathbf{a}_z \text{ mN}\end{aligned}$$

At that point,

$$\mathbf{E} = \frac{\mathbf{F}}{Q}$$

$$= (-6.507, -3.817, 7.506) \cdot \frac{10^{-3}}{10 \cdot 10^{-9}}$$

$$\mathbf{E} = -650.7\mathbf{a}_x - 381.7\mathbf{a}_y + 750.6\mathbf{a}_z \text{ kV/m}$$